The following homework contains topology concepts such as open, closed, and compact sets, as well as continuity. Feel free to work with each other. Please write your final submission on paper without lines. It is due during class **Thursday, January 23**, but is preferred by email beforehand.

Problem 1

Understand every step of every definition and proof shown in the course. If not, come to the final class with questions about anything you do not understand, or discuss with Ronak during normal lab hours. If everything is clear, then state on the homework "I understand everything from class perfectly."

Problem 2

Come up with one aspect of the course that was successful, and one aspect that could use improvement. Did the course mostly agree with or differ from your expectations?

Problem 1 and 2 Solution: To see a summary of the feedback, see class notes for January 23, 2020.

Problem 3

Prove that the intersection of two closed sets is closed.

Problem 3 Solution: Let $A, B \subseteq \mathbb{R}$ be closed sets. We wish to show that $A \cap B$ is closed. This can be done in two ways. The easy way is to show that $(A \cap B)^c = A^c \cup B^c$ is open. Becuase A^c and B^c are both open (as they are the complement of closed sets), their union is open (by a result from class). Thus, $A \cap B$, which is the complement of $(A \cap B)^c$, is closed.

The harder way is to let y be an AP of $A \cap B$. We know that for any $\epsilon > 0$, $(y - \epsilon, y + \epsilon)$ contains some $x \in A \cap B$ with $x \neq y$. This $x \in A$, so with ϵ arbitrary, y is an AP of A. Similarly, this $x \in B$, so with ϵ arbitrary, y is an AP of B. If y is an AP of A then it is in A, by the closure of A (same for B). If it is in A and B, it is in $A \cap B$ by definition. The result is shown.

Problem 4

If A is a set and f is a function, let $f(A) = \{f(x) : x \in A\}$, i.e. all outputs generated from inputs in A. Provide examples of the following. Provide examples of the following.

(a) Continuous f and open A such that $f(A)$ is not open.

(b) Continuous f and closed A such that $f(A)$ is not closed.

(c) Continuous f and compact $f(A)$ such that A is not compact.

The function and the sets need not be the same between examples, and you do not need to prove that the function is continuous or that the sets are open, closed, or compact.

Problem 4 Solution:

(a) $A = \mathbb{R}$, $f(x) = \sin(x)$, $f(A) = [-1, 1]$.

(b) $A = \mathbb{R}$, $f(x) = \arctan(x)$, $f(A) = (-\frac{\pi}{2}, \frac{\pi}{2})$.

(c) $A = \mathbb{R}$, $f(x) = \sin(x)$, $f(A) = [-1, 1]$.